A Computational Model of Higher-order Epistemic Reasoning

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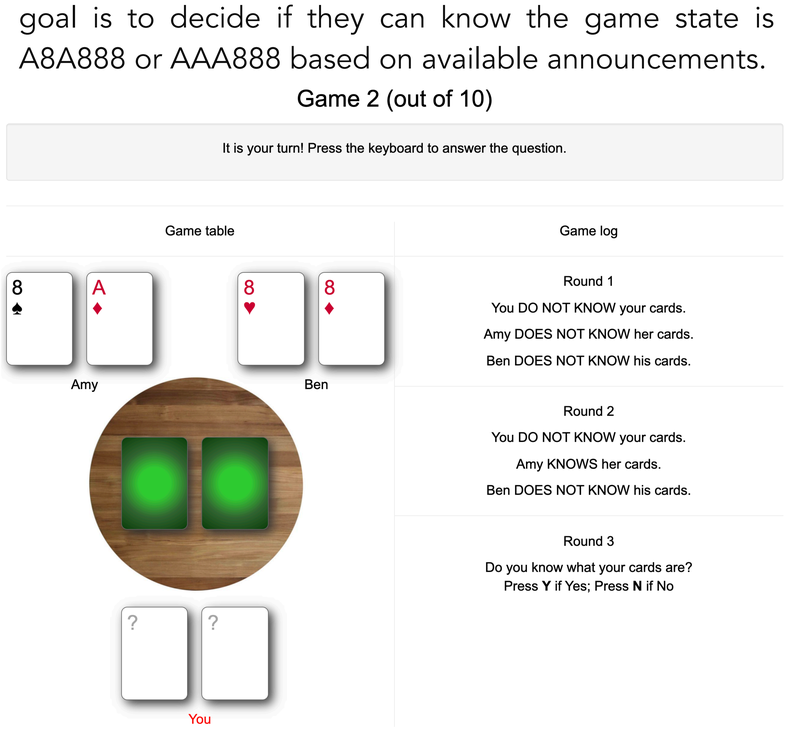
Introduction

Reasoning about what others know is prevalent in our daily life, known as **epistemic reasoning**. But sometimes we need to take one step further and **reason about how other people perform such reasoning**. Then what about two steps further or more? Inspired by **epistemic logic**,[1] we present a formal framework to capture how (and how much) people carry out higher-order epistemic reasoning in a deductive game called **Aces and Eights**.[2] In our framework, knowledge is modeled as **lack of uncertainty**.

Aces and Eights

In our task, participants play against **computer agents** Amy and Ben for 10 games. In each game, the 3 players each draw 2 cards from a deck of **4 Aces and 4 Eights**. All players **cannot see their own cards**. Following a predetermined order, players take turns to announce either **"I know my cards"** or **"I don't know my cards"**. We denote game states as "Participant's cards - Amy's cards - Ben's cards". In the example below, participants' goal is to decide if they can know the game state is A8A888 or AAA888 based on available announcements.



Represent Uncertainty

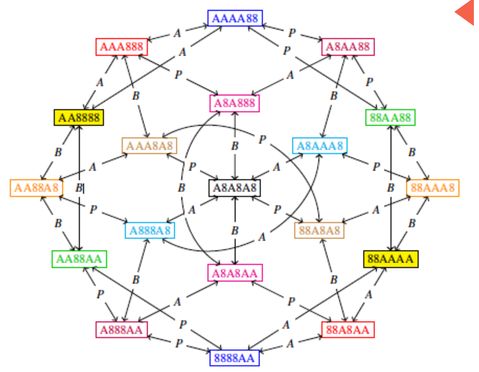


FIGURE 1: **Perfect reasoner**'s representation. Considers **all pairs of game states** (nodes) and **which players** (edge label) can't distinguish them. e.g., the participant, but not Amy or Ben, would find A8A888 possible if the true state is AAA888 and vice versa.

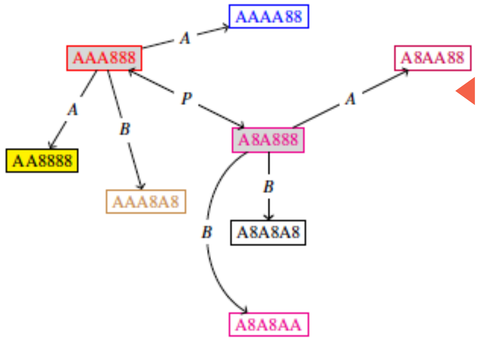
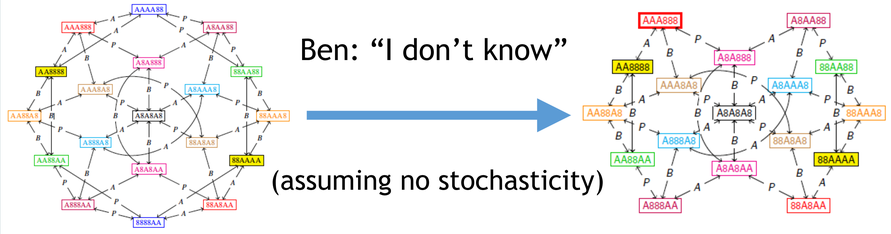


FIGURE 2: **Bounded reasoner**'s representation with epistemic level = 1. Only considers states reachable by **1 step** from the states possible to the participant before any announcement.

Following each announcement, uncertainty is **reduced by deleting nodes inconsistent with the announcement**. e.g.:



Candidate Models

Representation to action: Model makes announcements by either checking **if the participant still has uncertainty** or guessing with a certain probability.

Candidate models	Ignore others' announcement?	Fail to delete nodes?
With epistemic bound (-WEB)	SIWEB (possibly)	SUWEB (possibly)
With no epistemic bound (-WNB)	Noisy DEL (never)	SUWNB (possibly)

4 candidate models. **SUWEB** is the winning model. Columns are 2 different **sources of stochasticity**.

Results

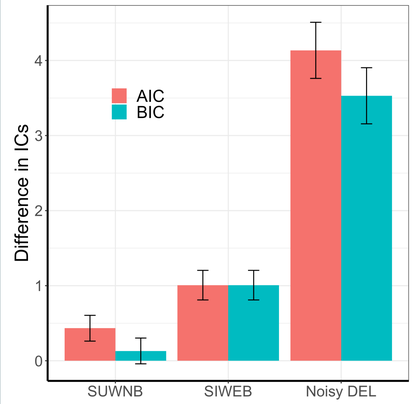


FIGURE 3: **Model comparison**. The AIC/BIC score of each model is subtracted by that of SUWEB. Error bars indicate the standard error of the mean.

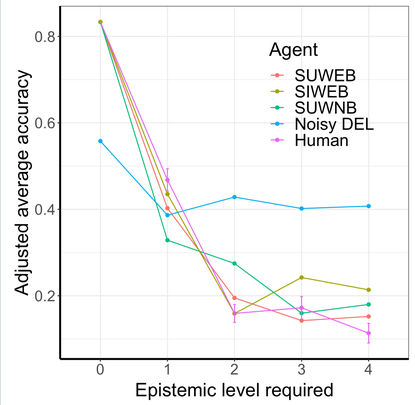


FIGURE 4: **Model validation**. The epistemic level required by a game is the minimum level for SUWEB to guarantee winning.

Summary

- This work proposed a logic-based framework that allows us to **formally model people's bounded capacity for higher-order epistemic reasoning** as epistemic level.
- Through model comparison, we provided evidence that epistemic level **contributes to predicting empirical data** beyond mere cognitive stochasticity.
- We pointed a new direction by showing what epistemic logic, or more generally **modal logic**, has to offer for cognitive science.

References

1. van Ditmarsch, H., Halpern, J., van der Hoek, W., & Kooi, B. (Eds.). (2015). *Handbook of epistemic logic*. College Publications.
2. Fagin, R., Halpern, J. Y., Moses, Y., & Vardi, M. Y. (2003). *Reasoning about knowledge*. MIT Press.

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